J. Fluid Mech. (1989), *vol.* 203, *pp.* 577–593
Printed in Great Britain

Influence of density variations on the structure of low-speed turbulent flows: a report on Euromech 237

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(Received 15 October 1988)

The European Mechanics Colloquium number 237 was held at the Institut de Mécanique Statistique de la Turbulence (Université d'Aix-Marseille II) from 18 to 21 July 1988. This was the first meeting to consider the influence of density variations on turbulent flows for both non-reacting and reacting flows in a variety of configurations. Several new experiments, computational models and theoretical analyses were presented for non-reacting flows. All these approaches showed the marked effects of density variations on coherent structures in turbulent shear flows. It was found that the approximate models used for turbulent Reynolds stresses in homogeneous flows do not need to be changed for the mean flow field but predictions for variances and correlations are still rather uncertain: in fact experimental results providing such information in variable-density flows are rare. The special problem of measuring turbulence in flows with density variations was discussed. In the discussions it was agreed that there are in fact strong similarities in the effects of density gradients on the dynamics of non-reacting flows and reacting flows despite the differences in the distributions of density gradients. Participants at the meeting from industry emphasized the importance of these flows to many different kinds of industrial and environmental problems.

1. Introduction

This Colloquium focused specifically on low-speed turbulent flows in which strong variation of density results from heat or mass transfer and from chemical reactions.

These turbulent flows are of fundamental interest because the conservation equations for thermodynamics, mass and momentum are linked together; in particular the effects of the density changes on the velocity and scalar fields cannot be neglected. The understanding of turbulent flows of strongly heated air discharging into cold air or turbulent flows of two gases which have global density differences are important problems in their own right. But this is also the key to an improved physical understanding of turbulent flows with combustion (Libby 1977) where the coupling between chemical reactions and aerodynamics occurs through the localized density fluctuations introduced by chemical heat release.

Flows with strong variations of density occur in many important practical situations: in the aerospace industry, in pollution and environmental problems (particularly in connection with dense gas releases to the atmosphere (Britter 1989) and with the many kinds of fire that occur in internal and external environments), in industrial heat exchangers, in carriage and storage of gases or liquids, in engine combustion chambers, and so on.

The understanding and modelling for constant-density turbulent flows is not yet by any means perfect. (For example, approximating correlations involving fluctuating gradients continues to be controversial in Reynolds stress modelling.) However, in many practical problems these uncertainties are smaller than those caused by variations in density. Despite a number of important papers on the effects of density variation on turbulent flows (e.g. Brown & Roshko 1974; Minh, Launder & MacInnes 1982; McMurty, Riley & Metcalfe 1989; Shih, Lumley & Janicka 1987), there has not so far been a conference especially devoted to this problem, which is surprising since variable-density flows are probably more numerous than constant-density flows as far as industrial applications are concerned.

One of the aims of this Colloquium was to allow an interaction of those researchers who specialize in turbulence of non-reacting flows with those who are involved in combustion. Another goal was to continue the work of the Euromech Colloquium (220) *Mixing and Chemical Reactions in Turbulent Flows* at Cambridge in March 1987, as well as to extend – on a European level – the French workshop organized by IMST on the same topics at Marseille in April 1985.

Euromech 237 was attended by about 60 participants from 13 different countries and 24 communications were presented. Two main subjects were considered: non-reacting flows and reacting flows. The first theme was more thoroughly investigated than the second and various topics were discussed: theoretical considerations, experimental and numerical results for mixing layers, jets, boundary layers, wakes and buoyancy effects. Papers on reacting flows were concerned with combusting flows, but comparisons were also made with non-reactive flows with density variations; both modelling and experimental approaches were considered here too.

2. Non-reacting flows – theoretical considerations

Formal perturbation analyses are often used as the starting point for turbulence models. H. Herwig (Institut für Thermo-und Fluiddynamik, Bochum) analysed the variable-density effect like other properties of the flow using a regular perturbation approach. He constructed a Taylor series expansion of all properties with respect to temperature and pressure where a small parameter is defined. The first effect of variable density is associated with the linear deviations with respect to this parameter. In particular the turbulent boundary layer was analysed, using the asymptotic two-layer structure of the flow. Rather than modelling the turbulence terms in the equations, a general functional form was assumed for the unknown distribution; and a few constants defining these functions are taken from experiments. The overall procedure was demonstrated for a fully developed turbulent pipe flow with a constant heat flux.

To help explain the modelling problem and some salient physical processes J. C. R. Hunt (University of Cambridge) also presented small-perturbation analyses (rapid-distortion theory) for velocity and density fluctuations in a flow with mean acceleration A parallel to the mean density gradient $\partial\bar{\rho}/\partial x$, and for a shear flow $U(y)$ perpendicular to the density gradient $\partial\bar{\rho}/\partial y$. These analyses were compared with those of fluid 'lumps' (spheres and cylinders) in the same flows, using a general inviscid equation for isolated rigid lumps with a density and a velocity different from those of the flow field (see Hunt 1987; Auton, Hunt & Prud'homme 1988; Rottman, Simpson & Stansby 1987). It was found that in stabilizing accelerating flows ($\boldsymbol{A} \cdot \nabla\bar{\rho} > 0$), the small-perturbation approach is physically appropriate and gives results similar to those obtained by calculating the motion of fluid lumps. But when the density perturbations are large ($\boldsymbol{A} \cdot \nabla\bar{\rho} < 0$), analysing the turbulence in terms of the motion of fluid lumps shows how the centripetal acceleration within large vortical structures concentrates the lighter fluid, as happens to bubbles in two-phase flows. In shear flows the small-perturbation analysis produces a weak effect of density fluctuations on Reynolds stresses, but considering fluid 'lumps' when analysing the nonlinear effects in a mean velocity gradient or in coherent structures shows how density effects can have strong effects, and that these are strongest when the turbulence structure and density distribution are rather non-Gaussian, such as in accelerating flame fronts.

The influence of density variations on turbulence spectra of heated flows was studied by C. Rey† & J. M. Rosant (Ecole Nationale Supérieure de Mécanique de Nantes). The equations are derived from classical Reynolds averaging and a first-order series expansion of pressure variations, involving a new set of equations for the velocity and temperature fluctuations. This set of equations can be used for the analysis of space–time correlations and spectra. Simplified equations were derived for the local isotropy range of the spectra when the Reynolds number is large. The density-variation effect appears to affect the small-scale structure of the turbulent velocity field less significantly than the small-scale structure of the temperature field, especially when the Prandtl number is large.

3. Non-reacting flows – mixing layers

In the two-dimensional mixing layer with a density difference across the layer, the density gradient persists along the length of the flow. This produces significant effects on the turbulence and so is a suitable geometry for experimental and computational studies of the effects. Nevertheless it has only received limited attention in the past although it is also an intriguing test case for turbulence modelling as well as of considerable technical interest. Among experimental works are the well-known paper by Brown & Roshko (1974) and those by Rebollo (1973) and Konrad (1976), and among numerical simulations is that by McMurtry *et al.* (1989).

Two connected papers from the Hermann-Föttinger-Institut (Technischen Universität Berlin) were presented. K. Nottmeyer† & H. E. Fiedler first showed experiments with a one-stream shear layer of several concentrations of helium in air. The spreading rate is evaluated from schlieren photographs: its increase with growing density differences implies an effect of large-scale structures, which in this situation become increasingly dominant. Velocity and density measurements are performed in a CO_2–air mixing layer with hot-wire probes. Two configurations were systematically studied: high velocity and high density in the same stream (cogradient), high velocity and low density in the same stream (countergradient). The spreading rate is decreased for the first case and increased for the second, which agrees with the results obtained using helium. The correlation coefficient between longitudinal velocity fluctuations, u', and density fluctuations, ρ', is completely different for the two cases: in the cogradient configuration this *correlation is positive*

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† Denotes the presenter of a multi-author paper.

and can reach a very high value (0.95) on the high-speed side; in the other configuration this correlation is positive on the low-speed side and negative on the high-speed side, the magnitude of the maximum values being about 0.6.

In the second paper a numerical simulation was presented by M. Lummer. He calculated the velocity and density fields of a variable-density mixing layer of air and helium similar to Brown & Roshko's experiments at Reynolds numbers up to 2710. He developed a fast computer code for the solution of the two-dimensional conservation equations in the vorticity–stream function formulation with periodic boundary conditions in the longitudinal direction. The case with lower density on the high-speed side shows a faster vortex pairing, indicating strong mixing between the two streams. The sign of the correlation between ρ' and the transverse velocity fluctuations, v', is always the opposite of that of the mean density gradient. The computations of the temporal spreading rates of cogradient and countergradient cases are nearly the same. There is no unique or certain way of converting these temporal growth rates into actual shear layers because the appropriate convection velocity is not known. Therefore, no direct comparison with previous experimental results can be made.

4. Non-reacting flows – jets

Though there have been relatively few measurements of the turbulent field behaviour in variable-density jets, they are more numerous than for mixing layers. Recently Pitts (1986) measured the effects of global density variations on mixing in turbulent axisymmetric jets, when the density ratio ranges from 0.14 up to 37. The spreading rate and the coherent structures were found to be markedly different over this range of conditions.

For convenience, jets in which heating causes density variations are considered first; and then those involving the mixing of two gases are examined. It can be assumed the effects on turbulence induced by these two types of density variations are similar.

4.1. *Heated jets*

F. Anselmet[†], R. Schiestel, E. M. Bahraoui, I. Zahibo & L. Fulachier (Institut de Mécanique Statistique de la Turbulence, Marseille) performed an experimental and numerical investigation of a vertical annular axisymmetric turbulent jet the inner part of which was heated by a laminar flame. Measurements and calculations are made above the flame where all chemical reactions are completed. It is also possible to heat slightly the outer air flow without combustion: heat is then a passive marker of the flow and can be used as a reference. The initial conditions for velocity and temperature are not quite similar in these two configurations, so that direct comparison can only be qualitative. The computations are based on second-order closures involving the full Reynolds stress transport equations, the turbulent heat flux equations, together with the equations for dissipation and variance of temperature fluctuations, using Favre averaging.[‡] The temperature dissipation is deduced from the ratio R of temperature and velocity dissipation timescales, which is usually assumed to be about to 0.5 (Béguier, Dekeyser & Launder 1978). The measured and computed characteristics of the mean flow fields are found to agree.

[‡] In Favre averaging (Favre 1969) the mean value of any quantity a is such that $\tilde{a} = \overline{\rho a}/\overline{\rho}$ and the fluctuation a' is such that $\overline{\rho a'} = 0$ with $a = \tilde{a} + a'$, while in Reynolds averaging $a = \bar{a} + a''$ with $\overline{a''} = 0$. The decomposition of ρ and pressure p are identical in the two methods ($\rho' = p' = 0$). The bar denotes the statistical mean value. See also §9.3.

The computed evolution of the axial temperature fluctuation variance is quite well predicted for the slightly heated flow with $R = 0.5$, but for the strongly heated case it is necessary to adjust this parameter to $R = 0.2$ for the computations to agree with the measurements: the ratio R seems to vary with density variations. This result can be related to that of Haroutunian, Ince & Launder (1988). Also, the measured centreline turbulent Prandtl number is found to be at least twice as large in the strongly heated flow as in the slightly heated one, which agrees with the results of the second-order model.

The stability and the growth of small disturbances in an axisymmetric heated jet discharging into cold stagnant air was investigated by P. A. Monkewitz† (University of California), D. W. Bechert & B. Barsikow (DFVLR, Technischen Universität, Berlin). They found that when the ratio S between the jet exit density, ρ_j, and ambient density, ρ_a, is less than 0.69, the character of the near-field pressure spectrum suddenly changes from a broadband spectrum to almost a line spectrum. There is a dominant line when $S = 0.5$. This suggests that for $S < 0.69$ the hot jet behaves like a self-excited oscillator, in good agreement with the stability studies of Huerre and Monkewitz (1985). When the Reynolds number is increased, keeping the density ratio fixed at, say, $S = 0.5$, the line spectrum reverts to a broadband form, at a Reynolds number of about 13000. Measurements beyond the initial laminar section also show that, whenever there is a unique mode to the spectrum, the jet is spreading at extraordinarily large half-angles of 30° and more. In the regimes with broad spectra, the spreading half-angle is in all cases very close to the usual 10°. This very large jet spreading is strongly intermittent and implies more entrainment and mixing than usual. This study shows that turbulence characteristics in free shear flows with variable density do not necessarily depend smoothly on the density ratio.

4.2. *Mixing of two different gases*

Using approximate models for the second-order moments of turbulence quantities, Shih *et al.* (1987) have recently argued that in jets and shear layers the turbulence is unaffected, to lowest order, by the density fluctuations, so that conventional models can be used for computing the mean flow field; the non-uniform density distribution would only influence the mean continuity and mean momentum equations. No experimental data currently exist to test these predictions in detail. Simultaneous measurements of variances, fluxes and third moments are needed. This is the objective of N. R. Panchapakesan & J. L. Lumley† (Cornell University, Ithaca), who are measuring (and modelling the flow using Shih *et al.* 1987) a helium jet discharging vertically into air. To minimize the problems of low flow speeds at the jet edges, they use 'flying' hot-wire measurements (Perry & Watmuff 1983). The main results are as follows: the spreading rate is twice as large as that for a homogeneous air jet and it is similar to that of a buoyant plume; the measured Reynolds stresses $\overline{u'v'}$ and $\overline{u'^2}$ are increased, while the prediction only shows small effects of ρ'; on the other hand, the measured lateral mass fraction flux $\overline{f'v'}$ is in good agreement with the prediction, which implies that density fluctuations are 'passively' produced, and that the eddy diffusivity is unchanged. Most of the structural differences in this variable-density jet seem to be buoyancy induced, even where initial jet momentum dominates buoyant momentum. This should be accounted for in modelling.

A similar jet study was described by P. Chassaing & M. Chibat† (Institut de Mécanique des Fluides, Toulouse), who also used the same Reynolds averaging as for homogeneous flows. In their case they measured the mixing of a CO_2 jet in still air

(Chassaing 1979). Their proposed model is a full second-order model, with a homogeneous dissipation ϵ equation. The dissipation rate of concentration fluctuations, ϵ_c, is modelled as usual but with $R = 0.7$. The results are quite good for the mean profiles, showing an improvement on previous calculations made with Favre averaging (Chassaing 1979). But an overprediction of 15–45% remains for the turbulence quantities, which is of the same order as with the previous Favre-based model.

W. P. Jones† & A. Pascau (Imperial College, London) have also applied a full second-order closure to a non-constant-density flow. Here, the flow configuration is more complicated: the study concerns a swirling jet in a duct, an experimental study that has also been carried out by So, Ahmed & Mongia (1984); the central jet is a CO_2 or He jet in order to emphasize the influence of density gradients of any sign in the presence of strong centrifugal forces. In their 'closure' they used Favre averaging, explicitly considered density effects in the pressure terms and took for the fluctuating pressure and transport terms those formulated by Jones & Musonge (1988) for constant-density flows. The only additional terms that appear owing to varying density are those involving the mean pressure gradient, and these do not represent a closure problem for pure mixing. Equations for ϵ and ϵ_c are also included in the model. The mean profiles are well represented, and the strong influence of the density gradient in the swirl is taken into account well. A similar order of magnitude discrepancy is found as for homogeneous flows between the second-order model and measurements for normal stresses.

D. Stepowski†, K. Labbaci, G. Cabot & M. Trinité (Faculté des Sciences, Rouen) investigated experimentally the pseudo-self-similarity in the development of a low-density vertical axisymmetric turbulent jet of hydrogen–nitrogen (equal parts by volume) issuing into coflowing air. Measurements of velocity and density are performed in the initial part of the mixing flow. The mean velocity is obtained by two-colour laser-Doppler anemometry and the mean concentration is deduced from measurements with the laser-Mie scattering technique where successively only the jet and then only the coflowing air are seeded. The influence of density variations on the pseudo-self-similar character of the jet is investigated using momentum and mass conservation arguments. In this connection the classical concept of an equivalent orifice diameter $d_{\mathrm{e}} = d(\rho_{\mathrm{j}}/\rho_{\mathrm{a}})^{\frac{1}{2}}$, where d is the jet diameter (see for instance Hinze 1959) is extended by considering at each section the mean density $\rho_{\mathrm{b}} = \int \rho u^2 \mathrm{d}s / \int u^2 \mathrm{d}s$, and a new equivalent orifice diameter $d'_{\mathrm{e}} = \mathrm{d}(\rho_{\mathrm{j}}/\rho_{\mathrm{b}})^{\frac{1}{2}}$ is defined. Mean velocity measurements confirm the pseudo-self-similarity character of the jet for $5 < X/d < 25$, where X is the longitudinal distance from the jet exit.

H. G. Green† & J. H. Whitelaw (Imperial College, London) had performed an experimental investigation of round jets of Freon-12, helium and air into still air in the near-field region. They used a combination of laser-Doppler velocimetry, laser-Rayleigh scattering and hot-wire anemometry to determine velocity and concentration characteristics. For a given Reynolds number, concentration decays and spreads more rapidly than velocity and the difference is greater in the high-density jet. The half-width of the scalar profile grows at a rate of 1.07 times that of velocity. Similar decay rates were obtained for jets of different densities provided they have a similar initial momentum flux. The asymptotic value of the mole fraction fluctuation intensity is reached more quickly in the high-density jet. Discrete frequencies in the near field are observed from the jet exit up to about seven diameters downstream, and they appear to be associated with vortex pairing; considering jets with an equal initial momentum flux, the classical Strouhal number

St varies from 0.14 to 0.8, but $St_\rho = St(\rho_a/\rho_j)^{\frac{1}{2}}$ is practically constant and equal to 0.38.

A research program has been developed by P. L. Viollett†, F. Lana, J. C. Olivier and D. Flamain (Electricité de France, Chatou) to compute the flow in an industrial turbulent jet that is discharged into colder surroundings by an electric arc heater (Lana & Violett 1987). In the particular flow studied there is a strongly heated jet (about 4000 K) discharging at 45° angle into the flow of a larger pipe (at about 1600 K). The main interest is to determine the jet deflection and the temperature of the opposite wall. In the experimental configuration the density variations are obtained with a jet of a mixture of helium and CO_2 discharging into a CO_2 flow. Mean concentration is measured with a continuous-sampling probe associated with a CO_2 analyser, and mean velocity with a hot-wire sensor. Computations, using Favre averaging, are performed in the three-dimensional configuration with the standard k–ϵ model where the turbulent Schmidt number is set equal to 1. The comparison between computations and experiments for both velocity and concentration is very satisfactory when the density ratio is close to 1. When the density ratio departs from unity and increases up to about 6, the penetration of the jet inside the pipe flow is underpredicted. Further work to investigate improvements in the turbulence model is in progress.

5. Non-reacting flows – boundary layers

Experimental studies in strongly heated turbulent boundary layers are scarce: the most significant one seems to be that of Cheng & Ng (1982). In this experiment the wall temperature is as high as 1250 K and the free-stream velocity is 10.5 m/s. G. Brun†, M. Buffat & D. Jeandel (Ecole Centrale de Lyon) presented a numerical model of this experiment, which includes Favre averaging and a k–ϵ model, but neglects any effect of the pressure gradient on the k and ϵ equations; a constant turbulent Prandtl number is assumed for the heat flux prediction. Computations mainly deal with the external region of the boundary layer, the region nearest the wall being approximated by so-called 'wall functions' (i.e. functions based on the Prandtl turbulent law of the wall). Although calculations predict reasonable mean profiles for velocity and temperature in both the isothermal and heated cases, the profiles of the turbulent stress and of the turbulent kinetic energy are quite far from the experiment results in the heated case. The value of the turbulent Prandtl number assumed in the model was varied between 0.9 and 1.5, but the agreement with the experiments was only slightly improved by taking the large value of 1.5; furthermore, results are found to be very sensitive to the choice of the wall functions used for the modelling of the log region of the flow.

R. Thünker† & W. Nitsche (Technischen Universität Berlin) investigated a heated or cooled turbulent flat-plate flow ($7.5 < U_{\text{Fluid}} < 20$ m/s and $-40 < T_{\text{Fluid}} < 200$ °C). The mean and fluctuating values of velocity and temperature are measured with a triple hot-wire probe. The local wall shear stress is determined by means of a sublayer fence and the local heat flux is obtained from the heat balance at the wall. These measurements are compared with numerical calculations based on a low-Reynolds-number k–ϵ turbulence model, with Favre averaging. The mean profiles agree well with the computation. Measured skin-friction coefficients and Stanton numbers, as function of the Reynolds number, are in good agreement with the homogeneous empirical function proposed for flows with small density variations. The measured Reynolds stress and, in particular, the turbulent heat flux are quite different from those computed. The measured turbulent Prandtl number is different from that usually obtained: this would be due in part to the density-variation effect and also to distinct thermal and dynamical boundary-layer thicknesses.

6. Non-reacting flow – wakes and buoyancy effects

The evolution of a Kármán vortex street in the presence of density inhomogeneities was presented in two papers. E. N. Ambartsoumian[†], G. S. Glushko & A. G. Gumilevski (Academy of Sciences, Moscow) investigated the characteristics of turbulence in the wake behind a circular cylinder, located in a horizontal gas flow with a vertical density gradient and a uniform velocity profile, in order to estimate quantitatively the influence of a stably stratified flow on the turbulent momentum transfer. The Reynolds number, based on the cylinder diameter and the free-stream velocity, is equal to 60, and the buoyancy period (the inverse of the Brünt–Väisälä frequency N) in the centre of the horizontal shear layer ranges from 0.66 up to 1 s, to be compared with an appropriate turbulent timescale. The Glushko's (1974) k–L model (L = macroscopic turbulent lengthscale) is extended for this horizontal shear flow with a vertically directed gravity force and stably stratified fluid. The turbulent viscosity ν_t is calculated numerically as a function of G and Z: $\nu_t = \alpha(G,Z)\,k^{\frac{1}{2}}L$, where $G = N^2L^2/k$ is the analogue of the Richardson number and Z is the non-dimensional gradient of mean velocity $U, Z = L(\partial U/\partial Y)/k^{\frac{1}{2}}$. Experimental results obtained using LDA indicate good agreement between theory and measurements. In particular α is found to be strongly dependent on the turbulent number G: α varies from 0.3 to 0.15 when G changes from 0 to 0.2.

In a second wake paper, D. Escudié (Ecole Centrale de Lyon) presented the interaction, over a limited region, between a Kármán vortex street in a vertical flow and another vertical flow with a large density gradient in the horizontal direction. This is created by a lean, premixed air–hydrogen laminar stabilized flame held on a very fine platinum catalytic wire of 0.4 mm diameter. The temperature of the burned gases is about 1200 K and the density ratio is equal to 3. The vortex street is generated by a rod (D = 1 mm diameter) located $10D$ upstream of the catalytic wire and parallel to it, at lateral distances of $4D$ or $8D$. Detailed measurements of the axis and transverse components of velocity determined by LDA were presented for three stations ($8D$, $15D$ and $80D$) downstream of the catalytic wire. In the constant-density case the wire and rod wakes are found to coalesce. In the presence of the density gradient, velocity measurements showed a competition between a deflection of the flame zone and shift of the Kármán vortex street. Fluctuating velocity measurements indicate that this interaction produces turbulence within the density gradient zone while no significant change is found within the Kármán vortex street itself.

Turbulent mixing flows with density variations at side weirs in rectangular water channels were examined by L. Yilmaz (Technical University of Istanbul). The energy equation is used in the description of the turbulent mixing of two phases with heat exchanges. Measurements of the hydrostatic pressure drop are compared with the theory.

7. Reacting flows – modelling

The problem of including the effects of variable and fluctuating density in the modelling of turbulent flows is probably most crucial for combusting flows. In contrast to the case of non-reacting flows, the use of Favre averaging is very widely adopted in the field of turbulent combustion. In fact, for some phenomena it is certainly not necessarily the best or only method. New models are needed and additional experiments in order to assess them. Three papers were devoted to the modelling problems for combusting flows.

First, a review was presented by R. Borghi (Faculté des Sciences, Rouen) in which the three aspects of modelling were separately discussed: the influence of density fluctuations on turbulent fluxes, on the turbulent kinetic energy, and on the scales and the dissipation rate of the turbulent kinetic energy. New terms related to correlations between pressure and velocity fluctuations were shown to be of great importance for these three aspects, especially in combusting flow. Only one of these terms is usually taken into account: it was shown to be related to the mean pressure gradient, and has the property of 'barodiffusion', that can apparently explain countergradient diffusion, as emphasized some years ago by Libby & Bray (1981). In some cases this effect can also be responsible for a large increase of the turbulent kinetic energy including larger anisotropy. A corresponding term, the correlation of the pressure fluctuation with the fluctuating divergence of velocity, has not been as well studied. Following the work of Kuznetzov (1979), it seems that it could be neglected in many cases, but not in premixed turbulent flames where the laminar flame speed is of the same order as the square root of the turbulent kinetic energy (Borghi 1985). The influence of such effects on the modelling of the turbulence scales, for instance through a dissipation rate equation, has to be necessarily taken into account, but it is far from being correctly understood at the present time.

The paper by P. Bruel† & M. Champion (Ecole Nationale Supérieure de Mécanique et d'Aéronautique, Poitiers) described the modelling procedure outlined by Borghi for a premixed turbulent flame stabilized in a boundary layer by distributed injection of hot gases through the wall. The model is written in the framework of very fast chemical reactions, so the mean combustion rate is controlled by the turbulent mixing. The calculations take into account the mean pressure gradient effect, and a full second-order modelling is proposed and discussed. The results of the computations first show the influence of the blowing through the wall and the combustion in the boundary layer on the transverse velocity profiles and the skin friction. The second-order closure is also compared with a classical gradient modelling technique and it is shown that the longitudinal diffusion flux strongly departs from the gradient assumption, while the lateral flux is more controlled by transverse gradients.

C. Bruno†, L. Galfetti & M. Belli (Universita di Milano) computed the flow in a typical industrial situation, namely, a flow with combustion heated behind a circular flame holder in a circular, cylindrical container. Using the widely known k–ϵ turbulence model, they considered cases with and without the modifications of density gradient proposed by Jones & McGuirk (1979). It was shown that apparently these modifications of the model, coupled with the nonlinearity of the combustion phenomena, can lead to strong differences in the predictions for the general flow field and for the temperature field. In some cases, ignition is predicted when density effects are considered that would not be achieved with the standard model. This was another example where a better basic understanding would be of significant industrial relevance.

8. Reacting flows – experiments

The experimental contributions presented at the workshop described new methods and gave new comparisons between combusting and non-combusting flows. A. Leipertz, G. Kowalewski† (Institut für thermo-und fluiddynamik, Bochum) & J. Haumann (BBC Research Center, Baden) discussed various methods of obtaining information about gas density structures by laser-Rayleigh scattering. Using this method in connection with correlation techniques they showed the possibility of determining temporally varying concentrations or temperature structures. They have extended the method to a planar imaging technique by forming a two-dimensional light sheet and using a two-dimensional photoelectrical detector. Instantaneous concentration and temperature measurements are obtained, providing qualitative visualization of the field as well as quantitative results with high temporal resolutions (lower than an ms) and spatial resolutions (lower than a mm^3). Experimental results concerning point measurements were presented for single and multiple quasi-parallel jet configurations (N_2 and CO_2, He–Ne and Ar) and in jets of hydrocarbon flames. Two-dimensional concentration and temperature measurements were also presented in cold free jets (methane jet in nitrogen, propane jet in nitrogen) and in the post-reaction zone of a laminar methane–air flame respectively.

Measurements in the near field of reacting and non-reacting complex jet flows were reported by D. F. G. Durao, M. V. Heitor† and A. L. N. Moreira (Instituto Superior Tecnico, Lisbon). This burner consists of a central axisymmetric swirling jet of 17 mm diameter surrounded by sixteen 6 mm circular jets, simulating the injection of oxygen in practical burners. The burner exits into a diverging tube (of fire brick) to improve the flame stability. LDA as well as thermocouple measurements were presented. Particular attention was paid by the authors to the error sources incurred in velocity measurements (suitability of seeding particles; it is necessary to have uniform concentrations of particles in the various jet flows as well as in the air entrained into the jet). Results were presented for central jet Reynolds numbers between 20000 and 45000, for central and peripherical jet velocities between 20 and 40 m/s, and for swirl numbers of the central jet (ratio of tangential to longitudinal exit velocities) of up to 2. Radial profiles of axial velocity, measured in isothermal and combusting flows, present the same general aerodynamic characteristics. Temperature measurements indicate that flame stabilization is achieved in an annular recirculation zone formed at the wall of the tube. Moreover, owing to the multiple jet configuration, turbulent mixing is mainly controlled by smaller-scale motions of the little jets rather than by larger motions on the scale of the tube. This may explain why turbulence measurements made in the vicinity of the exit of the tube show common features for the flows with and without combustion.

Variable-density effects in turbulent non-isothermal inert and turbulent reacting flows were presented by B. Sarh & I. Gökalp† (Centre de Recherches sur la Chimie de la Combustion et des Hautes Températures, Orléans). Experiments are carried out in rectangular vertical jets. In the inert flow, the density ratio between the heated jet and the ambient still air is varied down to 0.5, the exit Reynolds number being equal to 5000. In the reacting case, the investigation is conducted in a turbulent diffusion flame of fuel and air, the Reynolds number of which is 2400. In the same configuration, without a flame, the dynamic characteristics of this flow are studied at the same Reynolds number. The authors determined the velocity by laser-Doppler anemometry, the mean temperature in the flame by fine thermocouples, and, in the inert flows, temperature and heat fluxes were measured with fine wires and in combination with LDA. In the heated inert case, the influence of the density variation is stronger on the axial decay of the longitudinal mean velocity than on that of the mean temperature. The velocity and temperature half-widths are both slightly smaller in the strongly heated flow than in the constant-density one. The centreline velocity intensities slightly decrease with heating, while for temperature the decrease is much more pronounced. In the reacting case, the longitudinal velocity axial decay rate, the spreading rate and the centreline turbulence intensities are strongly reduced. A direct comparison of non-reactive flows with the diffusion flame is not possible owing to the changes of the flow field induced by heat release. This comparison should be attempted with the help of models and numerical computations.

9. Concluding remarks

In conclusion we identify three topics that have been particularly emphasized during this Colloquium, namely coherent structures, measurement techniques, and modelling.

9.1. *Coherent structures*

An interesting fundamental problem is whether, and if so how, density variations affect the large coherent structures that control the mixing in all turbulent shear flows. These structures can be loosely defined as regions of concentrated vorticity, with characteristic and flow-specific organization, and having recurrence, and an appreciable lifetime and scale (Fiedler 1988). They are responsible for about 20% of the turbulent energy in most of the flows and play an important role in the transport and entrainment mechanisms. Moreover it is found, in turbulent flows with a passive contaminant, that their contribution to the scalar variance and scalar fluxes is even more important (Antonia *et al.* 1987).

Since it appears that in most cases density variations affect turbulent fluxes and in particular Reynolds stresses, it seems obvious that coherent structures must also be modified. In free-shear layers, for instance, these structures tend to trap lumps of low-density fluid which significantly affects the transport properties of the layer, e.g. raising the eddy Schmidt number (Hunt *et al.* 1989) (cf. the paper presented by Anselmet *et al.*).

In modelling, it is difficult to take into account the role of coherent structures and it is necessary to correctly model at least third-order moments, since instabilities in the flow, organized motions and third-order moments appear to be linked together. Indeed second-order moments only give preliminary indications of the flow behaviour, and it is now necessary to go further. In fact the use of equations for moments requires an averaging procedure – either Favre, or Reynolds, or any other averaging – which is satisfactory neither in turbulent shear flows nor in flows where inhomogeneities are important, such as those involving density variations and especially combustion. Furthermore, more information than just the mean behaviour of the flow is required for industrial applications, as was pointed out in particular by G. Colenbrander (Shell laboratory, Amsterdam) during a Colloquium discussion. All the information needed is, in principle, contained in probability density funcions (p.d.f.s) of the appropriate variables; one can now measure these p.d.f.s, and modelling approaches to these are also in progress (Haworth & Pope 1987). But information more directly related to coherent structures can be gained by conditional-sampling analysis, and such an analysis appears to be of fundamental interest for measurements as well as for theoretical approaches. An experiment on a variable-density turbulent flow – for instance the mixing of two different gases – and making use of a passive scalar (by slightly heating or cooling the flow) to mark the structures, would be very useful. In addition, numerical simulations of density effects on these structures, taking into account their movement and their interactions, should be developed.

9.2. *Measurement techniques*

Progress in understanding low-speed turbulent flows with variable density is directly related to the quality of experimental techniques. In these flows where density variations can arise from non-homogeneities of composition or of temperature, it is particularly important to measure simultaneously velocity and scalars such as temperature and concentration.

In the case of non-reactive flows these measurements were achieved using techniques derived from hot-wire anemometry or by means of non-intrusive optical diagnostics. Concentration was measured by the probe described by Brown & Rebollo (1972) while concentration and velocity were determined simultaneously with the interfering hot-wire developed by Way & Libby (1970). A simple hot wire was also used and corrected for the influence of mean concentration. Calibration of such probes for use in turbulent flows of gas mixtures is often very time consuming. This can be overcome by using empirical laws so that a calibration of the wire response for a single gas would be sufficient to deduce the wire response for other gases and mixtures (Pitts & McCaffrey 1986). When density variations are related to large temperature fluctuations, the temperature measurements can be carried out using a very fine cold wire or a compensated fine wire thermocouple.

In reactive flows where high temperatures preclude the use of conventional velocity sensors, such as hot wires and Pitot tubes, laser-Doppler anemometry continues to be the most attractive technique for measuring velocity. It is worth noting the widespread use of the LDA in the case of non-reactive flows with density variations in those situations where the hot-wire calibration for gas mixtures is often tedious and difficult.

Some studies presented at the meeting have demonstrated the applicability of Rayleigh scattering to both non-reactive and reactive turbulent flows. Concentration measurements were restricted to binary gas mixtures in isothermal flow. On the other hand, temperature measurements were possible at constant gas pressure and for appropriate gas mixtures with nearly constant or calculable average Rayleigh cross-section. This point measurement technique can be extended to the planar imaging technique, providing quantitative measurements as well as qualitative visualizations. The interested reader may also refer to Long, Chu & Chang (1981) and Namazian, Schmitt & Long (1988) for more details. Mie-scattering measurements of mean concentration have also been cited for the development of a low-density turbulent jet mixing with coflowing air.

Other optical scattering techniques, not presented at the meeting, such as Raman scattering, CARS and laser-induced fluorescence can also be used to measure concentration and temperature (see Lapp & Penney 1981).

In non-reactive situations extended hot-wire anemometry and optical methods appear to be more complementary than competitive. For flux information involving intercorrelation between velocity and scalars or spectral measurements, extended hot-wire anemometry seems at this time more suitable than other methods. Two-dimensional mapping techniques, associated with digital imaging, are more useful for the determination of spatial scales and structures. In the case of reactive flows non-intrusive optical techniques have to be preferred. An appropriate use of such sophisticated techniques requires careful consideration of a wide variety of potential limitations (Dibble, Hartmann & Schefer 1987). It is worth noting that in some experiments optical techniques and thermocouple or cold-wire techniques have been combined (see for instance Samuelsen, La Rue & Seiler 1984; B. Sahr & I. Gökalp), but this attractive method still has problems to be overcome.

9.3. *Modelling*

The overall impression left by the modelling papers in this workshop appears quite encouraging for the future. The comparisons that have been described between experimental results and predictions seem to show that the present approaches are moving in the right direction. Four main points can be profitably emphasized.

(i) Both Favre averages and Reynolds averages have been used with similar success. Indeed, neither of these two approaches shows a clear advantage over the other; they both need a similar level of complexity in the modelling. The Reynolds-averaged terms possess clearer physical meanings, but the Favre-averaged terms allow more simplification of the conservation equations, especially in the case of combusting flows.

(ii) Good predictions can be achieved for the mean profiles but only with models including equations for the diffusion fluxes. Full second-order models behave generally quite well, but models based on an eddy diffusivity with given turbulent Prandtl or Schmidt numbers result in clear discrepancies. That finding is also true for complex flows with large pressure gradients, such as swirling flows. For these types of flows the terms associated with the mean and fluctuating pressure gradients can produce strong effects, that seem to be reasonably well predicted by the models so far. The improvements obtained by a second-order modelling are so evident, and the ability of this type of modelling to predict the effects of density differences in complex flows is so clear, that it is to be recommended that such model be used even for industrial purposes.

Predictions for variances and correlations are not quite as good as for mean profiles. In fact, experimental results that provide such information in non-constant-density flows are rare. There is a clear need for such results: good predictions of such quantities are of interest even if the mean profiles can be calculated with precision. In particular, for combusting flows, the turbulence characteristics, and especially timescales, can control the mean rate of consumption of reactants; in this case, a good prediction of these characteristics is absolutely necessary in order to get good profiles for mean quantities.

(iii) What appear to be the weak points in the models today? In our opinion, these are mainly two. First, there are the terms related to the fluctuating pressure gradient coupled with the density fluctuations (through temperature or species fluctuations). Such terms seem to be more easily handled with the usual Reynolds averaging, while they are never accounted for by those using Favre averaging. More insight about these quantities could be gained by comparing these two approaches in detail. This would be particularly useful for turbulent flows with premixed combustion, where a stronger influence of these terms is suspected. One can also recommend, in order to study these terms in more detail, the use of direct numerical simulations to gain information which cannot be measured.

The second weak point concerns the ϵ-equation. Essentially everything in this equation but the convection terms must be modelled, and the correct way in which to take into account the effects of density gradients and density fluctuations is unclear. Either with Reynolds averaging or with Favre averaging, the proposed equations used so far suffer from the lack of detailed justification. The proposals of Jones (1979), Vandromme (1980), Shih *et al.* (1987), are all different. Studies related to the influence of compression and expansion in reciprocating engines have also been faced with this ϵ-equation problem and the proposed solutions are also different (see for instance Morel & Mansour 1982).

Theoretical studies on the behaviour of velocity spectra, taking into account a velocity vector non-perpendicular to the wavenumber vector, as in variable-density flows, may be thought interesting in order to give more information on ϵ. Two-point turbulence models similar to the well-known EDQNM model could be attempted, in a way parallel to that used for incompressible turbulence. But J. Mathieu (Ecole Centrale de Lyon) reported at the Colloquium that such an approach leads to a 'quasi-untractable formulation'.

For a solution, one could equally recommend more detailed studies by means of direct numerical simulations, in the spirit of the recent study of Mansour, Kim & Moin (1987): the numerical simulation of McMurtry *et al.* (1989) could perhaps give interesting information, if the results were conveniently processed.

(iv) In this Colloquium the use of an equation for the destruction rate of the scalar fluctuations was either adopted or avoided in an equal number of papers. In passive scalar flows, it seems to be clear now that such an equation improves the predictions, although problems similar to those appearing for the building of an ϵ-equation always remain. More precisely, a larger set of different flow configurations can be handled by such an equation, used in conjunction with the second-order model. There is so far no information about the influence of density variations in this equation. In particular, as was discussed in this Colloquium, one may wonder whether a non-constant value of the classical ratio R of the scalar and the velocity dissipation timescales is to be expected.

Financial support for the colloquium was received from the following bodies: Commission of the European Community, Centre National de la Recherche Scientifique, Ministère de la Défense, Electricité de France, Gaz de France, Association Universitaire de Mécanique, Complexe de Recherche Interprofessionnel en Aérothermochimie (Université de Rouen), and Institut de Mécanique Statistique de la Turbulence (Université d'Aix-Marseille II). This support is gratefully acknowledged. We are especially grateful to Dr J. C. R. Hunt who animated the first general discussion and who has contributed much to the improvement of this report. We would also like to thank Professor J. L. Lumley for his general lecture, Professor W. P. Jones who animated the second general discussion, and Mrs C. Bernard, Dr M. P. Chauve, Chairman of the local organising committee, as well as the I.M.S.T. technical staff, who contributed to the success of the meeting.

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